$\mathsf{T_TT_2}$ with Termination Templates for Teaching

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Oxford

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Motivation

$$\begin{aligned} \mathsf{f}(x, \mathsf{g}(x)) &\to \mathsf{g}(\mathsf{f}(x, x)) \\ \mathsf{f}(x, x) &\to \mathsf{h}(\mathsf{g}(x)) \\ \mathsf{g}(\mathsf{f}(x, x)) &\to \mathsf{f}(\mathsf{h}(x), x) \end{aligned}$$

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Correct Termination Proofs?

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student 1:

$$w_0 = 1$$
$$\mathsf{w}(\mathsf{f}) = 4, \mathsf{w}(\mathsf{g}) = 2, \mathsf{w}(\mathsf{h}) = 1$$
$$\mathsf{h} > \mathsf{f} > \mathsf{g}$$

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student 2:

$$[\mathsf{h}](x) = x$$
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student 2:

$$[\mathsf{h}](x) = x$$
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student 3:

$$[\mathsf{h}](x) = \begin{pmatrix} 1 & 4 \\ 0 & 0 \end{pmatrix} x, \ [\mathsf{f}](x, y) = \begin{pmatrix} 1 & 15 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 16 & 1 \\ 0 & 0 \end{pmatrix} y + \begin{pmatrix} 8 \\ 0 \end{pmatrix},$$
$$[\mathsf{g}](x) = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Overview

- main idea
- $\mathsf{T_TT_2}$

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- $\mathsf{T_TT_2}$
- template mechanism
- URL encoding via the web interface

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Solve following Questions

- is the following termination proof correct?
- exists another termination proof with specific parameters?
- ...

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- based on SAT/SMT encodings

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- setting up examples during lecture is error-prone and tedious
- encode examples into URL
- automatically restore examples
- derive a convenient way to present examples

Template Mechanism for LPO, KBO, PIs and MIs

Addition on Natural Numbers

$$\mathsf{0} + y \to y$$
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LPO:

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Is there a precedence with

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How to call $\mathsf{T_TT_2}$

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Call with a Template

```
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```

Example: Full Call

```
./ttt2 -s 'poly -inters "+ = x0 + x1"' add.trs
```

Encoding in $\mathsf{T_TT_2}$

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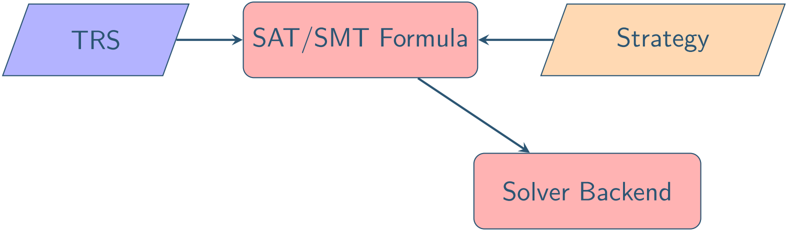

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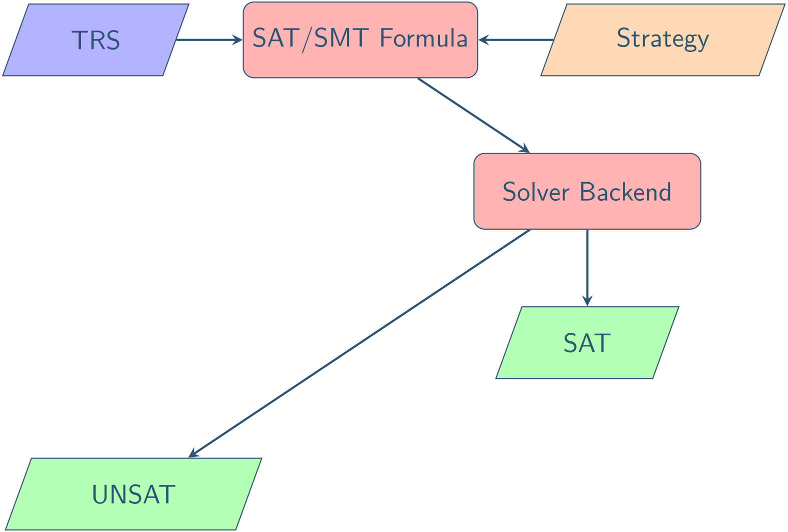

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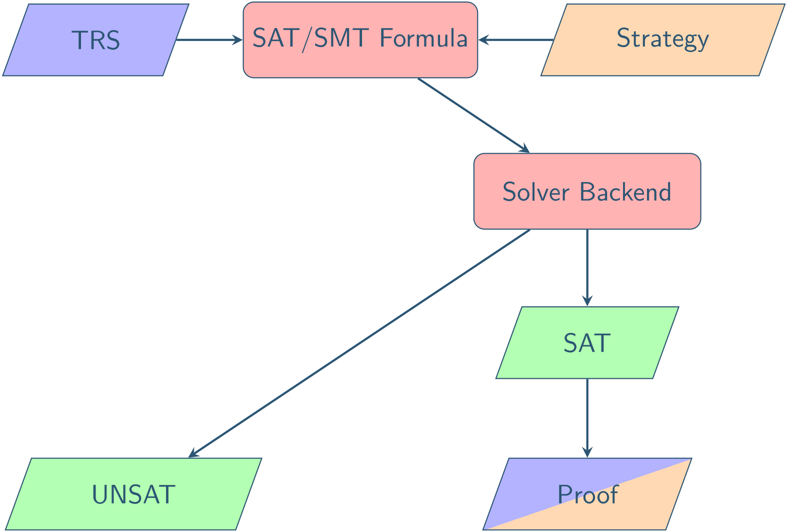

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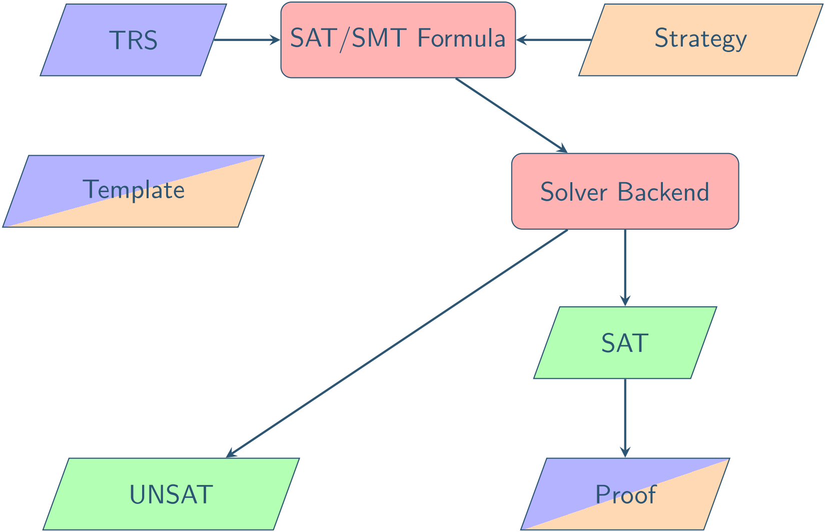

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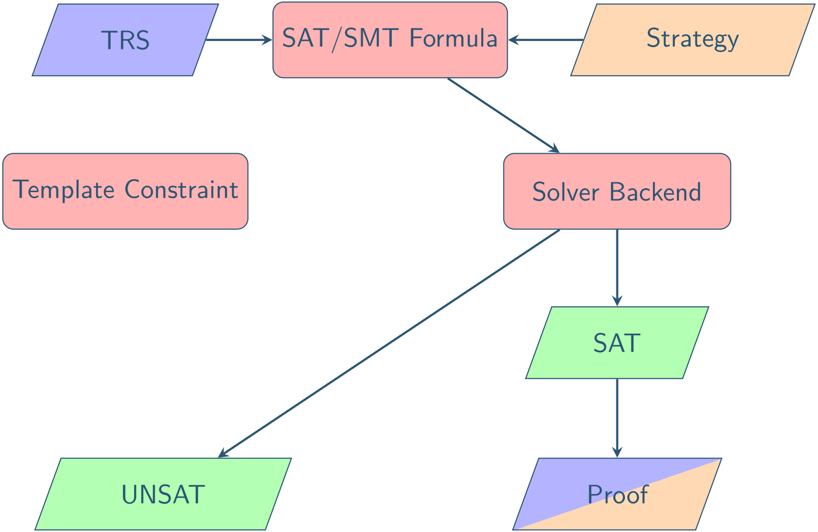

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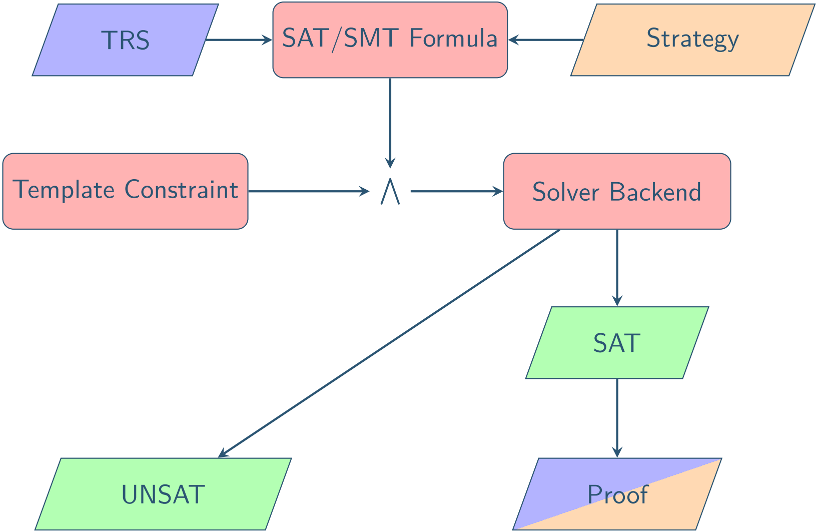

Lexicographic Path Order

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Syntax

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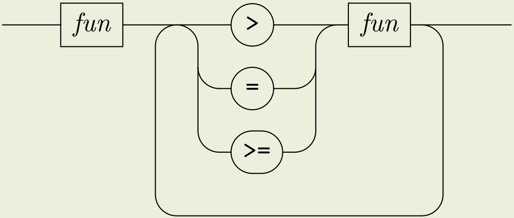

Addition on Natural Numbers

$$0 + y \to y$$
$$\mathsf{s}(x) + y \to \mathsf{s}(x + y)$$

Termination Proof without restrictions by $\mathsf{T_TT_2}$

precedence: $+ > \mathsf{s} \sim 0$

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```
./ttt2 -s 'lpo -prec"0 > +"' add.trs
```
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./ttt2 -s 'lpo -prec"+ > 0 > +"' add.trs    ✗
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./ttt2 -s 'lpo -prec"0 = + > s"' add.trs      ✓
```

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Knuth-Bendix Order

- parameters are precedence and weights

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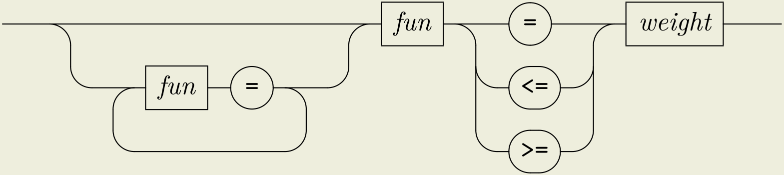
Syntax

weights



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```
./ttt2 -s 'kbo -weights "+ <= 16, + >= 8"' add.trs
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✓

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$$8 \leq \mathsf{w}(+) \leq 16$$

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$$16 \leq \mathsf{w}(+) \text{ and } \mathsf{w}(+) \leq 8$$

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$$0 = + > \mathsf{s}, \mathsf{w}(+) = 7, \mathsf{w}(\mathsf{s}) = 3 \text{ and } w_0 = 3$$

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```
./ttt2 -s 'kbo -prec "0 = + > s" -weights "+ = s = 3"
-w0 3' add.trs
```
✓

Linear Interpretations

- supported by PIs and MIs

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- accepted by the `-inters` flag

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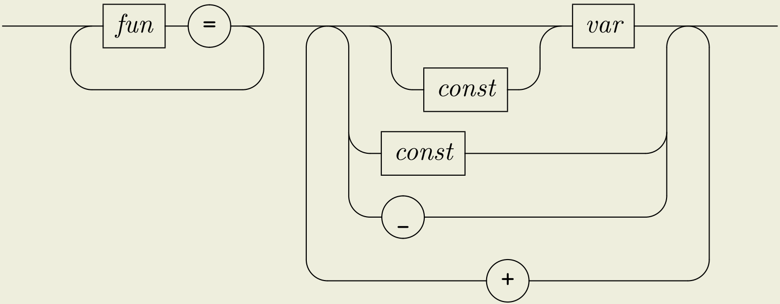
inters



Polynomial Interpretations

take natural numbers for “const” in the inters template

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Examples

$$[+](x, y) = 2x + y + 1, [\mathsf{s}](x) = x + 1, [0] = 0$$

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Examples

$$[+](x, y) = 2x + y + 1, [\mathsf{s}](x) = x + 1, [0] = 0$$

```
./ttt2 -s 'poly -inters"+ = 2x0 + x1 + 1, s = x0 + 1,
0 = 0"' add.trs
```
✓

Examples cont'd

$$[+](x, y) = 2x + ay + 1, [\mathsf{s}](x) = bx + c, [\mathsf{0}] = 0,$$

where a, b and c are arbitrary

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$$[+](x, y) = 2x + ay + 1, [\mathsf{s}](x) = bx + c, [\mathsf{0}] = 0,$$

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./ttt2 -s 'poly -inters "+ = 2x0 + _ + 0"' add.trs
```
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Matrix Interpretations

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- 0 and 1 for zero-vector and one-vector

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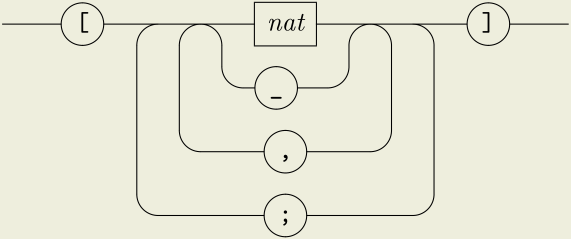
Syntax

matrix



Example: Syntax

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

in template syntax for MIs: `[1,2,3;4,5,6]`

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Examples

$$[0] = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad [\mathsf{s}](x) = x + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$[+](x, y) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} y + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Example: Syntax

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in template syntax for MIs: `[1,2,3;4,5,6]`

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```
./ttt2 -s 'matrix -inters "+ = [1,1;0,1]x0 + [1,0;0,1]x1
+ [1;0], s = x0 + 1, 0 = 0"' add.trs
```
✓

Examples cont'd

$$[0] = \begin{pmatrix} a \\ 0 \end{pmatrix} \qquad [\mathsf{s}](x) = x + m_1$$

$$[+](x, y) = \begin{pmatrix} 1 & b \\ c & d \end{pmatrix} x + m_2 y + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where a, b, c, d, m_1 and m_2 are arbitrary

Examples cont'd

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./ttt2 -s 'matrix -inters "+ = [1,_;_,_]x0 + _ + [1;0],
s = x0 + _, 0 = [_;0]' add.trs
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✓

Arbitrary Boolean Structure

- arbitrary boolean combinations of atomic constraints

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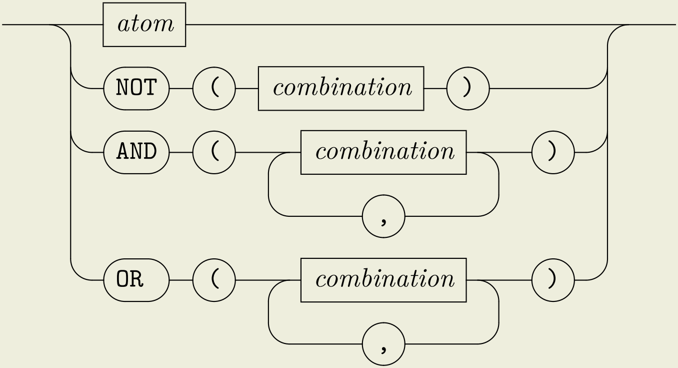
combination



Examples

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./ttt2 -s 'poly -inters "f=g=h=[1,_,_;0,1,_;0,0,1]x0+_,
g=h=[1,_,_;0,1,_;0,0,1]x1+_"' 27.xml
```
✓

Extension of the Web interface

Web Interface

Tyrolean Termination Tool 2 (1.17)

1. Input Term Rewrite System

For input use the standard TRS format.

select example ... ▾ or upload file Choose File No file chosen

```
(VAR x y)
(RULES
 add(0,y) -> y
 add(s(x),y) -> s(add(x,y))
 mul(0,y) -> 0
 mul(s(x),y) -> add(y,mul(x,y))
)
```

2. Select Strategy

◉ FAST ○ FBI ○ HYDRA ○ LPO ○ KBO ○ POLY ○ MATRIX(2) ○ MATRIX(3) ○ COMP ○ COMPLEXITY

○ EXPERT

3. Encode State into URL (optional)

encode URL | clear URL

4. Start TTT2

check ☐ use HTML output if available (*experimental feature*)

Web Interface - KBO

Tyrolean Termination Tool 2 (1.17)

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For input use the standard TRS format.

select example ... ▾ or upload file Choose File No file chosen

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 add(0,y) -> y
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○ EXPERT

KBO:

PRECEDENCE

WEIGHTS

W0

3. Encode State into URL (optional)

encode URL clear URL

4. Start TTT2

check ☐ use HTML output if available (*experimental feature*)

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URL with encoded Query String

http://colo6-c703.uibk.ac.at/ttt2/web/?problem=(VAR%20x%20y)%0A(RULES%0A%20add(0%2Cy)%20-%3E%20y%0A%20add(s(x)%2Cy)%20-%3E%20s(add(x%2Cy))%0A)&strategy=kbo&template=add%20%3E%20s&template1=add%20%3D%20s%20%3D%208&template2=3

Conclusion

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Future Work

- templates for other methods like arctic interpretations, the (generalized) subterm criterion, ...

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Future Work

- templates for other methods like arctic interpretations, the (generalized) subterm criterion, ...
- which extensions to our templates are most useful for teaching?

Thank you for your attention!